# A MECHANICAL ANALYSIS OF THE ERMI MPJ EXTENSIONATER ${ }^{\oplus}$ 

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## INTRODUCTION AND BACKGROUND

The MPJ Extensionater ${ }^{\text {² }}$ is a revolutionary device designed to increase the end range of motion at the first metatarsophalangeal joint. The machine attempts to reproduce the physiologic movement at this joint, placing the joint under tension in the same manner seen with normal ambulation. The rehabilitation protocol employs a variable position/variable load system, whereby the tension on the soft tissue increases as the boundaries of the end range of motion are encountered. A clinical investigation and mathematical study was constructed to determine if the MPJ Extensionater ${ }^{8}$ could produce adequate torque on the joint to stretch scar tissue. The results are presented along with a discussion.

## MECHANICAL ANALYSIS

When analyzing the MPJ Extensionater ${ }^{\text {² }}$, there are two very important aspects to be noticed. The first aspect is that the two moment arms connected to the toe joint bend upwards. The second aspect is that there are three forces contributing to the overall torque being applied at the toe joint. There is one force provided by the air cuff technology pushing downward right around the head of the metatarsal. This causes two reaction forces by the heel and toe to push upwards which ultimately causes the toe joint to bend. To calculate the overall range of torque, the Free Body Diagram (FBD) is used (Figure 1)

As seen in the FBD, there are three forces acting in the $y$-direction; we will call them $\mathrm{F}_{\text {neel }}$, $\mathrm{F}_{\text {lox }}$, and $F_{\text {airurft. }}$ The force equation would then be Fivel + $\mathrm{F}_{100}{ }^{*} \cos (\theta)=\mathrm{F}_{\text {airuff. }}$. We found that the air cuff technology could sustain roughly 11 psi over a bag surface area of $8 \mathrm{in}^{2}$. So, in theory if the entire air
cuff covered the top of the foot, it could potentially exert a maximum of 88 pounds of force. To continue our calculations, we made a few assumptions:

1. We assumed that the center of air cuff was consistently 1 inch away from the metatarsal toe joint on every body type.
2. We also assumed that the distance from the toe joint to the force where the toe was being exerted upward was 1.5 inches.
3. When the toe was being bent, the moment arm (heel to head of metatarsal) also bent to a certain degree. We assumed that the particular angle was negligible and we considered that angle to be $0^{\circ}$.
Therefore, in the next step of calculating the range of torque, we obtained the average foot lengths for children and adults. This information was compiled and coupled with a previous study done on the relationship between the length of the entire foot and the length from the heel to the head of the metatarsal (Ashford et al.: Austral J Pod Med 2002;36:45-9.). This ratio was found to be 1.45 ; we then applied this ratio to the obtained average foot lengths. In addition to using average dimensions of


Figure 1.
children and adults, we also used $15^{\circ}, 30^{\circ}, 45^{\circ}$, and $60^{\circ}$ as the amount of toe flexion, as they are the most common ranges of motion that we see with our patients.

To show how we obtained our numbers, we will use the dimensions of a child: 14.5 cm from the heel to the head of the metatarsal with toe flexion of $60^{\circ}$.

For our calculations, there were a few lengths that we needed. The first measured segment that we needed is denoted by the red line (distance from toe joint to center of the air cuff). Another segment that we needed is the perpendicular distance from the force at the toe to the heel (teal). This number was obtained by using the cosine function. The other segments used in the calculations were easy to obtain from the diagram.

To ensure the accuracy of our numbers, we also took into consideration the air cuff itself. Because the bag inflates into a round shape around the foot, not all $8 \mathrm{in} .^{2}$ of the bag will contribute to pushing the foot down. The bigger the foot, the more surface area the air cuff will cover the top of the leg and visa versa. So, after calculating the width of a child's foot, we approximated that $80 \%$ of the bag would be pushing downwards, or 6.4 in..$^{2}$. For a small adult, $85 \%$ or 6.8 in. $^{2}$, a medium adult, $90 \%$ or 7.2 in .2 , and lastly a large adult, $95 \%$ or 7.6 in. 2 .

Now that we have all the information we need to find out the maximum torque applied, we need to go back to our force equation: $\mathrm{F}_{\text {hed }}+\mathrm{F}_{\mathrm{Foc}}{ }^{*} \cos (60)$ $=$ Faruuff. We know the force that the air cuff exerts (dependent on body type), which leaves us two unknowns: the forces of the heel and toe. To find one of these forces, we need to do a sum of the torque forces about the heel. The formula for
determining torque is equal to "Force* Perpendicular Distance." The equation looks like the following:

$$
\begin{aligned}
& \sum\left(\mathrm{T} / \text { heel }=\left[-\left(\mathrm{F}_{\text {airuruf }}\right)^{*}(5.70186 \mathrm{in} .-1 \mathrm{in} .)\right]\right. \\
& \quad+\left[\left(\mathrm{F}_{\text {foce }}\right) *(2.85093 \mathrm{in.}+1.5 \mathrm{in} .)\right]=0
\end{aligned}
$$

To find the force the aircuff exerts on a child, it will be equal to: $11 \mathrm{psi}^{*} 6.4 \mathrm{in}^{2}=70.4$ pounds of force. Now, we only have one unknown in the equation, Ftoe. The force of the toe is 76.1 pounds. So from our equation $F_{\text {bec }}{ }^{*} \cos (60)+F_{\text {heel }}=F_{\text {aruruff }}$, we can deduce that the heel force is equal to 70.4 $76.1^{*} \cos (60)=32.36$ pounds. Now we can find the torque about the toe joint. The equation will look like the following:

$$
\begin{aligned}
& \sum\left(T / \text { ioc iomin }=\left[-\left(\mathrm{F}_{\text {aircuff }}\right)^{*}(1 \mathrm{in} .)\right]+\left[\left(\mathrm{F}_{\text {bece }}\right)^{*}(5.70186 \mathrm{in} .)\right]\right. \\
& +\left[\left(\mathrm{F}_{\mathrm{wc}}\right) *(1.5 \mathrm{in} .)\right]
\end{aligned}
$$

The maximum torque about the toe joint for a child with $60^{\circ}$ of toe flexion will be 19.02 footpounds. Using the corresponding values for adults of varying sizes in the equations, the results were tabulated as seen in Table 1.

## DISCUSSION

From our data, several interesting generalizations can be made. The data obtained in this study suggests that as the end range of motion is encountered, an increasing force is delivered behind the resistance; that is, when the joint reaches a point in its arc of motion that is the restriction point, more force is delivered at that point than had been applied to get to that point. Consequently, when force is needed to break through the restriction, it is available. We can see this as the logarithmic increase

Table 1

## MAXIMUM TORQUE VALUES.

## Length from Heel to Head of First Metatarsal (cm) Range of Torque (Foot-pounds)

$$
\begin{array}{ll}
\text { (Child) } 14.48 & 11.81\left(15^{\circ}\right), 12.85\left(30^{\circ}\right), 14.96\left(45^{\circ}\right), 19.02\left(60^{\circ}\right) \\
\text { (Adult S) } 15.86 & 13.02\left(15^{\circ}\right), 14.24\left(30^{\circ}\right), 16.58\left(45^{\circ}\right), 21.22\left(60^{\circ}\right) \\
\text { (Adult M) } 17.41 & 14.31\left(15^{\circ}\right), 15.59\left(30^{\circ}\right), 18.32\left(45^{\circ}\right), 23.53\left(60^{\circ}\right) \\
\text { (Adult L) } 18.97 & 15.51\left(15^{\circ}\right), 16.97\left(30^{\circ}\right), 19.94\left(45^{\circ}\right), 25.83\left(60^{\circ}\right)
\end{array}
$$

in torque as degrees of toe dorsi-flexion increase as identified in the diagram above. The increase is not only exponential in all cases but is proportional to the size foot the device is applied to. The data also suggests that the MPJ Extensionater ${ }^{8}$ more closely mimics the physiologic motion of the joint in terms of its ability to dynamically change the amount of torque as the flexion angle changes. Finally, on
comparative analysis, the torque generated by the MPJ Extensionater ${ }^{8}$ is far greater than most if not all products currently available for range of motion of the great toe joint. This tremendous torque is often required in extreme cases of limited joint motion where the scar tissue has matured and aggressive passive physical therapy cannot be sustained for long enough to stretch the scar tissue.

